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Three Dimensional Geometry

Short Answer Type Questions

Q. 1 Find the position vector of a point A in space such that $\vec{OA}$ is inclined at 60° to OX and at 45° to OY and $|\vec{OA}| = 10$ units.

Sol. Since, $\vec{OA}$ is inclined at 60° to OX and at 45° to OY . Let $\vec{OA}$ makes angle α with OZ .

$$\therefore \cos^2 60^\circ + \cos^2 45^\circ + \cos^2 \alpha = 1$$

$$\Rightarrow \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \cos^2 \alpha = 1 \quad [\because l^2 + m^2 + n^2 = 1]$$

$$\Rightarrow \frac{1}{4} + \frac{1}{2} + \cos^2 \alpha = 1$$

$$\Rightarrow \cos^2 \alpha = 1 - \left(\frac{1}{2} + \frac{1}{4}\right)$$

$$\Rightarrow \cos^2 \alpha = 1 - \left(\frac{6}{8}\right)$$

$$\Rightarrow \cos^2 \alpha = \frac{1}{4}$$

$$\Rightarrow \cos \alpha = \frac{1}{2} = \cos 60^\circ$$

$$\therefore \alpha = 60^\circ$$

$$\therefore \vec{OA} = |\vec{OA}| \left(\frac{1}{2} \hat{i} + \frac{1}{\sqrt{2}} \hat{j} + \frac{1}{2} \hat{k} \right)$$

$$= 10 \left(\frac{1}{2} \hat{i} + \frac{1}{\sqrt{2}} \hat{j} + \frac{1}{2} \hat{k} \right) \quad [\because |\vec{OA}| = 10]$$

$$= 5\hat{i} + 5\sqrt{2}\hat{j} + 5\hat{k}$$



Q. 2 Find the vector equation of the line which is parallel to the vector $3\hat{i} - 2\hat{j} + 6\hat{k}$ and which passes through the point $(1, -2, 3)$.

Thinking Process

Here, we use the formula $\vec{r} = \vec{b} + \lambda \vec{a}$, where $\vec{r}$ is the equation of the line which passes through $\vec{b}$ and parallel to $\vec{a}$.

Sol. Let $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$

So, vector equation of the line, which is parallel to the vector $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$ and passes through the vector $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ is $\vec{r} = \vec{b} + \lambda \vec{a}$.

$$\therefore \vec{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(3\hat{i} - 2\hat{j} + 6\hat{k})$$

$$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) - (\hat{i} - 2\hat{j} + 3\hat{k}) = \lambda(3\hat{i} - 2\hat{j} + 6\hat{k})$$

$$\Rightarrow (x-1)\hat{i} + (y+2)\hat{j} + (z-3)\hat{k} = \lambda(3\hat{i} - 2\hat{j} + 6\hat{k})$$

Q. 3 Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect.

Also, find their point of intersection.

Thinking Process

If shortest distance between the lines is zero, then they intersect.

Sol. We have, $x_1 = 1, y_1 = 2, z_1 = 3$ and $a_1 = 2, b_1 = 3, c_1 = 4$

Also, $x_2 = 4, y_2 = 1, z_2 = 0$ and $a_2 = 5, b_2 = 2, c_2 = 1$

If two lines intersect, then shortest distance between them should be zero.

$\therefore$ Shortest distance between two given lines

$$\begin{aligned} & \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2 + (a_1b_2 - a_2b_1)^2}} \\ &= \frac{\begin{vmatrix} 4-1 & 1-2 & 0-3 \\ 2 & 3 & 4 \\ 5 & 2 & 1 \end{vmatrix}}{\sqrt{(3 \cdot 1 - 2 \cdot 4)^2 + (4 \cdot 5 - 1 \cdot 2)^2 + (2 \cdot 2 - 5 \cdot 3)^2}} \\ &= \frac{\begin{vmatrix} 3 & -1 & -3 \\ 2 & 3 & 4 \\ 5 & 2 & 1 \end{vmatrix}}{\sqrt{25 + 324 + 121}} \\ &= \frac{3(3-8) + 1(2-20) - 3(4-15)}{\sqrt{470}} \\ &= \frac{-15 - 18 + 33}{\sqrt{470}} = \frac{0}{\sqrt{470}} = 0 \end{aligned}$$

Therefore, the given two lines are intersecting.

For finding their point of intersection for first line,

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$$

$$\Rightarrow x = 2\lambda + 1, y = 3\lambda + 2 \text{ and } z = 4\lambda + 3$$

Since, the lines are intersecting. So, let us put these values in the equation of another line.

$$\text{Thus, } \frac{2\lambda + 1 - 4}{5} = \frac{3\lambda + 2 - 1}{2} = \frac{4\lambda + 3}{1}$$

$$\Rightarrow \frac{2\lambda - 3}{5} = \frac{3\lambda + 1}{2} = \frac{4\lambda + 3}{1}$$

$$\Rightarrow \frac{2\lambda - 3}{5} = \frac{4\lambda + 3}{1}$$

$$\Rightarrow 2\lambda - 3 = 20\lambda + 15$$

$$\Rightarrow 18\lambda = -18 = -1$$

So, the required point of intersection is

$$x = 2(-1) + 1 = -1$$

$$y = 3(-1) + 2 = -1$$

$$z = 4(-1) + 3 = -1$$

Thus, the lines intersect at $(-1, -1, -1)$.

Q. 4 Find the angle between the lines

$$\vec{r} = 3\hat{i} - 2\hat{j} + 6\hat{k} + \lambda(2\hat{i} + \hat{j} + 2\hat{k}) \text{ and } \vec{r} = (2\hat{j} - 5\hat{k}) + \mu(6\hat{i} + 3\hat{j} + 2\hat{k}).$$

Thinking Process

We know that, $\cos \theta = \frac{|\vec{b}_1 \cdot \vec{b}_2|}{|\vec{b}_1| \cdot |\vec{b}_2|}$, where, θ is the angle between the lines $\vec{a}_1 + \lambda \vec{b}_1$

and $\vec{a}_2 + \mu \vec{b}_2$.

Sol. We have,

$$\vec{r} = 3\hat{i} - 2\hat{j} + 6\hat{k} + \lambda(2\hat{i} + \hat{j} + 2\hat{k})$$

and

$$\vec{r} = (2\hat{j} - 5\hat{k}) + \mu(6\hat{i} + 3\hat{j} + 2\hat{k})$$

where,

$$\vec{a}_1 = 3\hat{i} - 2\hat{j} + 6\hat{k}, \vec{b}_1 = 2\hat{i} + \hat{j} + 2\hat{k}$$

and

$$\vec{a}_2 = 2\hat{j} - 5\hat{k}, \vec{b}_2 = 6\hat{i} + 3\hat{j} + 2\hat{k}$$

If θ is angle between the lines, then

$$\begin{aligned} \cos \theta &= \frac{|\vec{b}_1 \cdot \vec{b}_2|}{|\vec{b}_1| \cdot |\vec{b}_2|} \\ &= \frac{|(2\hat{i} + \hat{j} + 2\hat{k}) \cdot (6\hat{i} + 3\hat{j} + 2\hat{k})|}{|2\hat{i} + \hat{j} + 2\hat{k}| |6\hat{i} + 3\hat{j} + 2\hat{k}|} \\ &= \frac{|12 + 3 + 4|}{\sqrt{9} \sqrt{49}} = \frac{19}{21} \\ \therefore \theta &= \cos^{-1} \frac{19}{21} \end{aligned}$$

Q. 5 Prove that the line through $A(0, -1, -1)$ and $B(4, 5, 1)$ intersects the line through $C(3, 9, 4)$ and $D(-4, 4, 4)$.

Sol. We know that, the cartesian equation of a line that passes through two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

Hence, the cartesian equation of line passes through $A(0, -1, -1)$ and $B(4, 5, 1)$ is

$$\begin{aligned} \frac{x - 0}{4 - 0} &= \frac{y + 1}{5 + 1} = \frac{z + 1}{1 + 1} \\ \Rightarrow \frac{x}{4} &= \frac{y + 1}{6} = \frac{z + 1}{2} \end{aligned} \quad \dots(i)$$

and cartesian equation of the line passes through $C(3, 9, 4)$ and $D(-4, 4, 4)$ is

$$\begin{aligned} \frac{x - 3}{-4 - 3} &= \frac{y - 9}{4 - 9} = \frac{z - 4}{4 - 4} \\ \Rightarrow \frac{x - 3}{-7} &= \frac{y - 9}{-5} = \frac{z - 4}{0} \end{aligned} \quad \dots(ii)$$

If the lines intersect, then shortest distance between both of them should be zero.

$\therefore$ Shortest distance between the lines

$$\begin{aligned} & \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2 + (a_1b_2 - a_2b_1)^2}} \\ &= \frac{\begin{vmatrix} 3 - 0 & 9 + 1 & 4 + 1 \\ 4 & 6 & 2 \\ -7 & -5 & 0 \end{vmatrix}}{\sqrt{(6 \cdot 0 + 10)^2 + (-14 - 0)^2 + (-20 + 42)^2}} \\ &= \frac{\begin{vmatrix} 3 & 10 & 5 \\ 4 & 6 & 2 \\ -7 & -5 & 0 \end{vmatrix}}{\sqrt{100 + 196 + 484}} \\ &= \frac{3(0 + 10) - 10(14) + 5(-20 + 42)}{\sqrt{780}} \\ &= \frac{30 - 140 + 110}{\sqrt{780}} = 0 \end{aligned}$$

So, the given lines intersect.

Q. 6 Prove that the lines $x = py + q$, $z = ry + s$ and $x = p'y + q'$, $z = r'y + s'$ are perpendicular, if $pp' + rr' + 1 = 0$.

Sol. We have, $x = py + q \Rightarrow y = \frac{x - q}{p}$...(i)

and $z = ry + s \Rightarrow y = \frac{z - s}{r}$...(ii)

$\Rightarrow \frac{x - q}{p} = \frac{y}{1} = \frac{z - s}{r}$ [using Eqs. (i) and (ii)] ... (iii)

Similarly, $\frac{x - q'}{p'} = \frac{y}{1} = \frac{z - s'}{r'}$...(iv)

From Eqs. (iii) and (iv),

$$a_1 = p, b_1 = 1, c_1 = r$$

and

$$a_2 = p', b_2 = 1, c_2 = r'$$

If these given lines are perpendicular to each other, then

$$a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$$

$\Rightarrow$

$$pp' + 1 + rr' = 0$$

which is the required condition.

Q. 7 Find the equation of a plane which bisects perpendicularly the line joining the points $A(2, 3, 4)$ and $B(4, 5, 8)$ at right angles.

Sol. Since, the equation of a plane is bisecting perpendicular the line joining the points $A(2, 3, 4)$ and $B(4, 5, 8)$ at right angles.

So, mid-point of AB is $\left(\frac{2+4}{2}, \frac{3+5}{2}, \frac{4+8}{2}\right)$ i.e., $(3, 4, 6)$.

Also,
$$\vec{N} = (4-2)\hat{i} + (5-3)\hat{j} + (8-4)\hat{k} = 2\hat{i} + 2\hat{j} + 4\hat{k}$$

So, the required equation of the plane is $(\vec{r} - \vec{a}) \cdot \vec{N} = 0$.

$$\Rightarrow [(x-3)\hat{i} + (y-4)\hat{j} + (z-6)\hat{k}] \cdot (2\hat{i} + 2\hat{j} + 4\hat{k}) = 0 \quad [\because \vec{a} = 3\hat{i} + 4\hat{j} + 6\hat{k}]$$

$$\Rightarrow 2x - 6 + 2y - 8 + 4z - 24 = 0$$

$$\Rightarrow 2x + 2y + 4z = 38$$

$$\therefore x + y + 2z = 19$$

Q. 8 Find the equation of a plane which is at a distance $3\sqrt{3}$ units from origin and the normal to which is equally inclined to coordinate axis.

Sol. Since, normal to the plane is equally inclined to the coordinate axis.

Therefore,
$$\cos \alpha = \cos \beta = \cos \gamma = \frac{1}{\sqrt{3}}$$

So, the normal is $\vec{N} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$ and plane is at a distance of $3\sqrt{3}$ units from origin.

The equation of plane is $\vec{r} \cdot \hat{N} = 3\sqrt{3}$

$$\left[\because \hat{N} = \frac{\vec{N}}{|\vec{N}|} \right]$$

[since, vector equation of the plane at a distance p from the origin is $\vec{r} \cdot \hat{N} = p$]

$$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) \cdot \frac{\left(\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}\right)}{1} = 3\sqrt{3}$$

$$\Rightarrow \frac{x}{\sqrt{3}} + \frac{y}{\sqrt{3}} + \frac{z}{\sqrt{3}} = 3\sqrt{3}$$

$$\therefore x + y + z = 3\sqrt{3} \cdot \sqrt{3} = 9$$

So, the required equation of plane is $x + y + z = 9$.



Q. 9 If the line drawn from the point $(-2, -1, -3)$ meets a plane at right angle at the point $(1, -3, 3)$, then find the equation of the plane.

Sol. Since, the line drawn from the point $(-2, -1, -3)$ meets a plane at right angle at the point $(1, -3, 3)$. So, the plane passes through the point $(1, -3, 3)$ and normal to plane is $(-3\hat{i} + 2\hat{j} - 6\hat{k})$.

$$\Rightarrow \vec{a} = \hat{i} - 3\hat{j} + 3\hat{k}$$

$$\text{and } \vec{N} = -3\hat{i} + 2\hat{j} - 6\hat{k}$$

So, the equation of required plane is $(\vec{r} - \vec{a}) \cdot \vec{N} = 0$

$$\Rightarrow [(x\hat{i} + y\hat{j} + z\hat{k}) - (\hat{i} - 3\hat{j} + 3\hat{k})] \cdot (-3\hat{i} + 2\hat{j} - 6\hat{k}) = 0$$

$$\Rightarrow [(x-1)\hat{i} + (y+3)\hat{j} + (z-3)\hat{k}] \cdot (-3\hat{i} + 2\hat{j} - 6\hat{k}) = 0$$

$$\Rightarrow -3x + 3 + 2y + 6 - 6z + 18 = 0$$

$$\Rightarrow -3x + 2y - 6z = -27$$

$$\therefore 3x - 2y + 6z - 27 = 0$$

Q. 10 Find the equation of the plane through the points $(2, 1, 0)$, $(3, -2, -2)$ and $(3, 1, 7)$.

Thinking Process

Here, apply the equation of the plane passing through the points (x_1, y_1, z_1) , (x_2, y_2, z_2)

$$\text{and } (x_3, y_3, z_3) \text{ is given by } \begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0.$$

Sol. We know that, the equation of a plane passing through three non-collinear points (x_1, y_1, z_1) , (x_2, y_2, z_2) and (x_3, y_3, z_3) is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-2 & y-1 & z-0 \\ 3-2 & -2-1 & -2-0 \\ 3-2 & 1-1 & 7-0 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-2 & y-1 & z \\ 1 & -3 & -2 \\ 1 & 0 & 7 \end{vmatrix} = 0$$

$$\Rightarrow (x-2)(-21+0) - (y-1)(7+2) + z(3) = 0$$

$$\Rightarrow -21x + 42 - 9y + 9 + 3z = 0$$

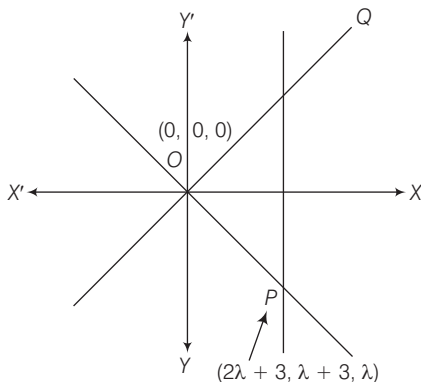
$$\Rightarrow -21x - 9y + 3z = -51$$

$$\therefore 7x + 3y - z = 17$$

So, the required equation of plane is $7x + 3y - z = 17$.

Q. 11 Find the equations of the two lines through the origin which intersect the line $\frac{x-3}{2} = \frac{y-3}{1} = \frac{z}{1}$ at angles of $\frac{\pi}{3}$ each.

Sol. Given equation of the line is $\frac{x-3}{2} = \frac{y-3}{1} = \frac{z}{1} = \lambda$... (i)



So, DR's of the line are 2, 1, 1 and DC's of the given line are $\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{16}}$.

Also, the required lines make angle $\frac{\pi}{3}$ with the given line.

From Eq. (i), $x = (2\lambda + 3)$, $y = (\lambda + 3)$ and $z = \lambda$

$$\therefore \cos \theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$\therefore \cos \frac{\pi}{3} = \frac{(4\lambda + 6) + (\lambda + 3) + (\lambda)}{\sqrt{6} \sqrt{(2\lambda + 3)^2 + (\lambda + 3)^2 + \lambda^2}}$$

$$\Rightarrow \frac{1}{2} = \frac{6\lambda + 9}{\sqrt{6} \sqrt{(4\lambda^2 + 9 + 12\lambda + \lambda^2 + 9 + 6\lambda + \lambda^2)}}$$

$$\Rightarrow \frac{\sqrt{6}}{2} = \frac{6\lambda + 9}{\sqrt{6\lambda^2 + 18\lambda + 18}}$$

$$\Rightarrow 6\sqrt{(\lambda^2 + 3\lambda + 3)} = 2(6\lambda + 9)$$

$$\Rightarrow 36(\lambda^2 + 3\lambda + 3) = 36(4\lambda^2 + 9 + 12\lambda)$$

$$\Rightarrow \lambda^2 + 3\lambda + 3 = 4\lambda^2 + 9 + 12\lambda$$

$$\Rightarrow 3\lambda^2 + 9\lambda + 6 = 0$$

$$\Rightarrow \lambda^2 + 3\lambda + 2 = 0$$

$$\Rightarrow \lambda(\lambda + 2) + 1(\lambda + 2) = 0$$

$$\Rightarrow (\lambda + 1)(\lambda + 2) = 0$$

$$\therefore \lambda = -1, -2$$

So, the DC's are 1, 2, -1 and -1, 1, -2.

Also, both the required lines pass through origin.

So, the equations of required lines are $\frac{x}{1} = \frac{y}{2} = \frac{z}{-1}$ and $\frac{x}{-1} = \frac{y}{1} = \frac{z}{-2}$.

Q. 12 Find the angle between the lines whose direction cosines are given by the equation $l + m + n = 0$ and $l^2 + m^2 - n^2 = 0$.

Sol. Eliminating n from both the equations, we have

$$\begin{aligned} l^2 + m^2 - (l - m)^2 &= 0 \\ \Rightarrow l^2 + m^2 - l^2 - m^2 + 2ml &= 0 \Rightarrow 2lm = 0 \\ \Rightarrow lm &= 0 \Rightarrow (-m - n)m = 0 \quad [\because l = -m - n] \\ \Rightarrow (m + n)m &= 0 \\ \Rightarrow m &= -n \Rightarrow m = 0 \\ \Rightarrow l &= 0, l = -n \end{aligned}$$

Thus, Dir's two lines are proportional to $0, -n, n$ and $-n, 0, n$ i.e., $0, -1, 1$ and $-1, 0, 1$.

So, the vector parallel to these given lines are $\vec{a} = -\hat{j} + \hat{k}$ and $\vec{b} = -\hat{i} + \hat{k}$

Now,
$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \Rightarrow \cos \theta = \frac{1}{2}$$

$$\therefore \theta = \frac{\pi}{3} \quad \left[\because \cos \frac{\pi}{3} = \frac{1}{2} \right]$$

Q. 13 If a variable line in two adjacent positions has direction cosines l, m, n and $l + \delta l, m + \delta m, n + \delta n$, then show that the small angle $\delta \theta$ between the two positions is given by $\delta \theta^2 = \delta l^2 + \delta m^2 + \delta n^2$.

Sol. We have l, m, n and $l + \delta l, m + \delta m, n + \delta n$ as direction cosines of a variable line in two different positions.

$$\therefore l^2 + m^2 + n^2 = 1 \quad \dots (i)$$

$$\text{and } (l + \delta l)^2 + (m + \delta m)^2 + (n + \delta n)^2 = 1 \quad \dots (ii)$$

$$\Rightarrow l^2 + m^2 + n^2 + \delta l^2 + \delta m^2 + \delta n^2 + 2(l \delta l + m \delta m + n \delta n) = 1$$

$$\Rightarrow \delta l^2 + \delta m^2 + \delta n^2 = -2(l \delta l + m \delta m + n \delta n) \quad [\because l^2 + m^2 + n^2 = 1]$$

$$\Rightarrow l \delta l + m \delta m + n \delta n = \frac{-1}{2}(\delta l^2 + \delta m^2 + \delta n^2) \quad \dots (iii)$$

Now, $\vec{a}$ and $\vec{b}$ are unit vectors along a line with direction cosines l, m, n and $(l + \delta l), (m + \delta m), (n + \delta n)$, respectively.

$$\therefore \vec{a} = l\hat{i} + m\hat{j} + n\hat{k} \text{ and } \vec{b} = (l + \delta l)\hat{i} + (m + \delta m)\hat{j} + (n + \delta n)\hat{k}$$

$$\Rightarrow \cos \delta \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \vec{a} \cdot \vec{b} \quad [\because |\vec{a}| = |\vec{b}| = 1]$$

$$\begin{aligned} \Rightarrow \cos \delta \theta &= l(l + \delta l) + m(m + \delta m) + n(n + \delta n) \\ &= (l^2 + m^2 + n^2) + (l \delta l + m \delta m + n \delta n) \\ &= 1 - \frac{1}{2}(\delta l^2 + \delta m^2 + \delta n^2) \quad [\text{using Eq. (iii)}] \end{aligned}$$

$$\Rightarrow 2(1 - \cos \delta \theta) = (\delta l^2 + \delta m^2 + \delta n^2)$$

$$\Rightarrow 2 \cdot 2 \sin^2 \frac{\delta \theta}{2} = \delta l^2 + \delta m^2 + \delta n^2 \quad \left[\because 1 - \cos \theta = 2 \sin^2 \frac{\theta}{2} \right]$$

$$\Rightarrow 4 \left(\frac{\delta \theta}{2} \right)^2 = \delta l^2 + \delta m^2 + \delta n^2 \quad \left[\text{since, } \frac{\delta \theta}{2} \text{ is small, then } \sin \frac{\delta \theta}{2} = \frac{\delta \theta}{2} \right]$$

$$\therefore \delta \theta^2 = \delta l^2 + \delta m^2 + \delta n^2$$

Q. 14 If O is the origin and A is (a, b, c) , then find the direction cosines of the line OA and the equation of plane through A at right angle to OA .

Sol. Since, DC's of line OA are $\frac{a}{\sqrt{a^2 + b^2 + c^2}}$, $\frac{b}{\sqrt{a^2 + b^2 + c^2}}$ and $\frac{c}{\sqrt{a^2 + b^2 + c^2}}$.

Also, $\vec{n} = \vec{OA} = \vec{a} = a\hat{i} + b\hat{j} + c\hat{k}$

The equation of plane passes through (a, b, c) and perpendicular to OA is given by

$$\begin{aligned} & [\vec{r} - \vec{a}] \cdot \vec{n} = 0 \\ \Rightarrow & \vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n} \\ \Rightarrow & (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (a\hat{i} + b\hat{j} + c\hat{k}) = (a\hat{i} + b\hat{j} + c\hat{k}) \cdot (a\hat{i} + b\hat{j} + c\hat{k}) \\ \Rightarrow & ax + by + cz = a^2 + b^2 + c^2 \end{aligned}$$

Q. 15 Two systems of rectangular axis have the same origin. If a plane cuts them at distances a, b, c and a', b', c' , respectively from the origin, then prove that $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2}$.

Sol. Consider OX, OY, OZ and ox, oy, oz are two system of rectangular axes. Let their corresponding equation of plane be

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \quad \dots(i)$$

$$\text{and} \quad \frac{x}{a'} + \frac{y}{b'} + \frac{z}{c'} = 1 \quad \dots(ii)$$

Also, the length of perpendicular from origin to Eqs. (i) and (ii) must be same.

$$\begin{aligned} \therefore & \frac{\frac{0}{a} + \frac{0}{b} + \frac{0}{c} - 1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}} = \frac{\frac{0}{a'} + \frac{0}{b'} + \frac{0}{c'} - 1}{\sqrt{\frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2}}} \\ \Rightarrow & \sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}} = \sqrt{\frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2}} \\ \Rightarrow & \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2} \end{aligned}$$

Long Answer Type Questions

Q. 16 Find the foot of perpendicular from the point $(2, 3, -8)$ to the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$. Also, find the perpendicular distance from the given point to the line.

Sol. We have, equation of line as $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$

$$\Rightarrow \frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3} = \lambda$$

$$\Rightarrow x = -2\lambda + 4, y = 6\lambda \text{ and } z = -3\lambda + 1$$

Let the coordinates of L be $(4 - 2\lambda, 6\lambda, 1 - 3\lambda)$ and direction ratios of PL are proportional to $(4 - 2\lambda - 2, 6\lambda - 3, 1 - 3\lambda + 8)$ i.e., $(2 - 2\lambda, 6\lambda - 3, 9 - 3\lambda)$.

Also, direction ratios are proportional to $-2, 6, -3$. Since, PL is perpendicular to given line.

$$\therefore -2(2 - 2\lambda) + 6(6\lambda - 3) - 3(9 - 3\lambda) = 0$$

$$\Rightarrow -4 + 4\lambda + 36\lambda - 18 - 27 + 9\lambda = 0$$

$$\Rightarrow 49\lambda = 49 \Rightarrow \lambda = 1$$

So, the coordinates of L are $(4 - 2\lambda, 6\lambda, 1 - 3\lambda)$ i.e., $(2, 6, -2)$.

$$\begin{array}{c} P(2, 3, -8) \\ | \\ L \\ \hline \frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3} \end{array}$$

$$\begin{aligned} \text{Also, length of } PL &= \sqrt{(2-2)^2 + (6-3)^2 + (-2+8)^2} \\ &= \sqrt{0+9+36} = 3\sqrt{5} \text{ units} \end{aligned}$$

Q. 17 Find the distance of a point $(2, 4, -1)$ from the line

$$\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}.$$

Sol. We have, equation of the line as $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9} = \lambda$

$$\Rightarrow x = \lambda - 5, y = 4\lambda - 3, z = 6 - 9\lambda$$

Let the coordinates of L be $(\lambda - 5, 4\lambda - 3, 6 - 9\lambda)$, then Dr's of PL are $(\lambda - 7, 4\lambda - 7, 7 - 9\lambda)$.

Also, the direction ratios of given line are proportional to $1, 4, -9$.

Since, PL is perpendicular to the given line.

$$\therefore (\lambda - 7) \cdot 1 + (4\lambda - 7) \cdot 4 + (7 - 9\lambda) \cdot (-9) = 0$$

$$\Rightarrow \lambda - 7 + 16\lambda - 28 + 81\lambda - 63 = 0$$

$$\Rightarrow 98\lambda = 98 \Rightarrow \lambda = 1$$

So, the coordinates of L are $(-4, 1, -3)$.

$$\begin{aligned} \therefore \text{Required distance, } PL &= \sqrt{(-4-2)^2 + (1-4)^2 + (-3+1)^2} \\ &= \sqrt{36+9+4} = 7 \text{ units} \end{aligned}$$

Q. 18 Find the length and the foot of perpendicular from the point $\left(1, \frac{3}{2}, 2\right)$ to the plane $2x - 2y + 4z + 5 = 0$.

Sol. Equation of the given plane is $2x - 2y + 4z + 5 = 0$... (i)

$$\Rightarrow \vec{n} = 2\hat{i} - 2\hat{j} + 4\hat{k}$$

So, the equation of line through $\left(1, \frac{3}{2}, 2\right)$ and parallel to $\vec{n}$ is given by

$$\frac{x-1}{2} = \frac{y-\frac{3}{2}}{-2} = \frac{z-2}{4} = \lambda$$

$$\Rightarrow x = 2\lambda + 1, y = -2\lambda + \frac{3}{2} \text{ and } z = 4\lambda + 2$$

If this point lies on the given plane, then

$$2(2\lambda + 1) - 2\left(-2\lambda + \frac{3}{2}\right) + 4(4\lambda + 2) + 5 = 0 \quad [\text{using Eq. (i)}]$$

$$\Rightarrow 4\lambda + 2 + 4\lambda - 3 + 16\lambda + 8 + 5 = 0$$

$$\Rightarrow 24\lambda = -12 \Rightarrow \lambda = \frac{-1}{2}$$

$\therefore$ Required foot of perpendicular

$$= \left[2 \times \left(\frac{-1}{2}\right) + 1, -2 \times \left(\frac{-1}{2}\right) + \frac{3}{2}, 4 \times \left(\frac{-1}{2}\right) + 2 \right] \text{ i.e., } \left(0, \frac{5}{2}, 0\right)$$

$$\therefore \text{ Required length of perpendicular} = \sqrt{(1-0)^2 + \left(\frac{3}{2} - \frac{5}{2}\right)^2 + (2-0)^2}$$

$$= \sqrt{1 + 1 + 4} = \sqrt{6} \text{ units}$$

Q. 19 Find the equation of the line passing through the point $(3, 0, 1)$ and parallel to the planes $x + 2y = 0$ and $3y - z = 0$.

Sol. Equation of the two planes are $x + 2y = 0$ and $3y - z = 0$.

Let $\vec{n}_1$ and $\vec{n}_2$ are the normals to the two planes, respectively.

$$\therefore \vec{n}_1 = \hat{i} + 2\hat{j} \quad \text{and} \quad \vec{n}_2 = 3\hat{j} - \hat{k}$$

Since, required line is parallel to the given two planes.

$$\text{Therefore, } \vec{b} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 0 \\ 0 & 3 & -1 \end{vmatrix}$$

$$= \hat{i}(-2) - \hat{j}(-1) + \hat{k}(3)$$

$$= -2\hat{i} + \hat{j} + 3\hat{k}$$

So, the equation of the lines through the point $(3, 0, 1)$ and parallel to the given two planes are

$$(x-3)\hat{i} + (y-0)\hat{j} + (z-1)\hat{k} + \lambda(-2\hat{i} + \hat{j} + 3\hat{k})$$

$$\Rightarrow (x-3)\hat{i} + y\hat{j} + (z-1)\hat{k} + \lambda(-2\hat{i} + \hat{j} + 3\hat{k})$$

Q. 20 Find the equation of the plane through the points $(2, 1, -1)$, $(-1, 3, 4)$ and perpendicular to the plane $x - 2y + 4z = 10$.

Sol. The equation of the plane passing through $(2, 1, -1)$ is

$$a(x-2) + b(y-1) + c(z+1) = 0 \quad \dots(i)$$

Since, this passes through $(-1, 3, 4)$.

$$\therefore a(-1-2) + b(3-1) + c(4+1) = 0$$

$$\Rightarrow -3a + 2b + 5c = 0 \quad \dots(ii)$$

Since, the plane (i) is perpendicular to the plane $x - 2y + 4z = 10$.

$$\therefore 1 \cdot a - 2 \cdot b + 4 \cdot c = 0$$

$$\Rightarrow a - 2b + 4c = 0 \quad \dots(iii)$$

On solving Eqs. (ii) and (iii), we get

$$\frac{a}{8+10} = \frac{-b}{-17} = \frac{c}{4} = \lambda$$

$$\Rightarrow a = 18\lambda, b = 17\lambda, c = 4\lambda$$

From Eq. (i),

$$\begin{aligned} 18\lambda(x-2) + 17\lambda(y-1) + 4\lambda(z+1) &= 0 \\ \Rightarrow 18x - 36 + 17y - 17 + 4z + 4 &= 0 \\ \Rightarrow 18x + 17y + 4z - 49 &= 0 \\ \therefore 18x + 17y + 4z &= 49 \end{aligned}$$

Q. 21 Find the shortest distance between the lines gives by

$$\vec{r} = (8 + 3\lambda)\hat{i} - (9 + 16\lambda)\hat{j} + (10 + 7\lambda)\hat{k}$$

and

$$\vec{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k}).$$

Sol. We have,

$$\begin{aligned} \vec{r} &= (8 + 3\lambda)\hat{i} - (9 + 16\lambda)\hat{j} + (10 + 7\lambda)\hat{k} \\ &= 8\hat{i} - 9\hat{j} + 10\hat{k} + 3\lambda\hat{i} - 16\lambda\hat{j} + 7\lambda\hat{k} \\ &= 8\hat{i} - 9\hat{j} + 10\hat{k} + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k}) \end{aligned}$$

$$\Rightarrow \vec{a}_1 = 8\hat{i} - 9\hat{j} + 10\hat{k} \text{ and } \vec{b}_1 = 3\hat{i} - 16\hat{j} + 7\hat{k} \quad \dots(i)$$

$$\text{Also } \vec{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$$

$$\Rightarrow \vec{a}_2 = 15\hat{i} + 29\hat{j} + 5\hat{k} \text{ and } \vec{b}_2 = 3\hat{i} + 8\hat{j} - 5\hat{k} \quad \dots(ii)$$

$$\text{Now, shortest distance between two lines is given by } \left| \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$$\begin{aligned} \therefore \vec{b}_1 \times \vec{b}_2 &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix} \\ &= \hat{i}(80 - 56) - \hat{j}(-15 - 21) + \hat{k}(24 + 48) \\ &= 24\hat{i} + 36\hat{j} + 72\hat{k} \end{aligned}$$

$$\begin{aligned} \text{Now, } |\vec{b}_1 \times \vec{b}_2| &= \sqrt{(24)^2 + (36)^2 + (72)^2} \\ &= 12\sqrt{2^2 + 3^2 + 6^2} = 84 \end{aligned}$$

$$\begin{aligned} \text{and } (\vec{a}_2 - \vec{a}_1) &= (15 - 8)\hat{i} + (29 + 9)\hat{j} + (5 - 10)\hat{k} \\ &= 7\hat{i} + 38\hat{j} - 5\hat{k} \end{aligned}$$

$$\begin{aligned} \therefore \text{Shortest distance} &= \left| \frac{(24\hat{i} + 36\hat{j} + 72\hat{k}) \cdot (7\hat{i} + 38\hat{j} - 5\hat{k})}{84} \right| \\ &= \left| \frac{168 + 1368 - 360}{84} \right| = \left| \frac{1176}{84} \right| = 14 \text{ units} \end{aligned}$$

Q. 22 Find the equation of the plane which is perpendicular to the plane $5x + 3y + 6z + 8 = 0$ and which contains the line of intersection of the planes $x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$.

Sol. The equation of a plane through the line of intersection of the planes $x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$ is

$$\begin{aligned} (x + 2y + 3z - 4) + \lambda(2x + y - z + 5) &= 0 \\ \Rightarrow x(1 + 2\lambda) + y(2 + \lambda) + z(-\lambda + 3) - 4 + 5\lambda &= 0 \quad \dots(i) \end{aligned}$$

Also, this is perpendicular to the plane $5x + 3y + 6z + 8 = 0$.

$$\therefore 5(1 + 2\lambda) + 3(2 + \lambda) + 6(3 - \lambda) = 0$$

$$[\because a_1 a_2 + b_1 b_2 + c_1 c_2 = 0]$$

$$\Rightarrow 5 + 10\lambda + 6 + 3\lambda + 18 - 6\lambda = 0$$

$$\therefore \lambda = -29/7$$

From Eq. (i),

$$x \left[1 + 2 \left(\frac{-29}{7} \right) \right] + y \left(2 - \frac{29}{7} \right) + z \left(\frac{29}{7} + 3 \right) - 4 + 5 \left(\frac{-29}{7} \right) = 0$$

$$\Rightarrow x(7 - 58) + y(14 - 29) + z(29 + 21) - 28 - 145 = 0$$

$$\Rightarrow -51x - 15y + 50z - 173 = 0$$

So, the required equation of plane is $51x + 15y - 50z + 173 = 0$.

Q. 23 If the plane $ax + by = 0$ is rotated about its line of intersection with the plane $z = 0$ through an angle α , then prove that the equation of the plane in its new position is $ax + by \pm (\sqrt{a^2 + b^2} \tan \alpha) z = 0$.

Sol. Equation of the plane is $ax + by = 0$

... (i)

$\therefore$ Equation of the plane after new position is

$$\frac{ax \cos \alpha}{\sqrt{a^2 + b^2}} + \frac{by \cos \alpha}{\sqrt{b^2 + a^2}} \pm z \sin \alpha = 0$$

$$\Rightarrow \frac{ax}{\sqrt{a^2 + b^2}} + \frac{by}{\sqrt{b^2 + a^2}} \pm z \tan \alpha = 0 \quad [\text{on dividing by } \cos \alpha]$$

$$\Rightarrow ax + by \pm z \tan \alpha \sqrt{a^2 + b^2} = 0 \quad [\text{on multiplying with } \sqrt{a^2 + b^2}]$$

Alternate Method

Given, planes are

$$ax + by = 0 \quad \dots (i)$$

and

$$z = 0 \quad \dots (ii)$$

Therefore, the equation of any plane passing through the line of intersection of planes

(i) and (ii) may be taken as $ax + by + k = 0$.

... (iii)

Then, direction cosines of a normal to the plane (iii) are $\frac{a}{\sqrt{a^2 + b^2 + k^2}}, \frac{b}{\sqrt{a^2 + b^2 + k^2}},$

$\frac{c}{\sqrt{a^2 + b^2 + k^2}}$ and direction cosines of the normal to the plane (i) are $\frac{a}{\sqrt{a^2 + b^2}}, \frac{b}{\sqrt{a^2 + b^2}},$

0.

Since, the angle between the planes (i) and (ii) is α ,

$$\therefore \cos \alpha = \frac{a \cdot a + b \cdot b + k \cdot 0}{\sqrt{a^2 + b^2 + k^2} \sqrt{a^2 + b^2}}$$

$$= \frac{\sqrt{a^2 + b^2}}{\sqrt{a^2 + b^2 + k^2}}$$

$$\Rightarrow k^2 \cos^2 \alpha = a^2 (1 - \cos^2 \alpha) + b^2 (1 - \cos^2 \alpha)$$

$$\Rightarrow k^2 = \frac{(a^2 + b^2) \sin^2 \alpha}{\cos^2 \alpha}$$

$$k = \pm \sqrt{a^2 + b^2} \tan \alpha$$

On putting this value in plane (iii), we get the equation of the plane as

$$ax + by + z \sqrt{a^2 + b^2} \tan \alpha = 0$$

Q. 24 Find the equation of the plane through the intersection of the planes $\vec{r} \cdot (\hat{i} + 3\hat{j}) - 6 = 0$ and $\vec{r} \cdot (3\hat{i} - \hat{j} - 4\hat{k}) = 0$, whose perpendicular distance from origin is unity.

Sol. We have, $\vec{n}_1 = (\hat{i} + 3\hat{j})$, $d_1 = 6$ and $\vec{n}_2 = (3\hat{i} - \hat{j} - 4\hat{k})$, $d_2 = 0$

Using the relation, $\vec{r} \cdot (\vec{n}_1 + \lambda \vec{n}_2) = d_1 + d_2 \lambda$

$$\Rightarrow \vec{r} \cdot [(\hat{i} + 3\hat{j}) + \lambda(3\hat{i} - \hat{j} - 4\hat{k})] = 6 + 0 \cdot \lambda$$

$$\Rightarrow \vec{r} \cdot [(1 + 3\lambda)\hat{i} + (3 - \lambda)\hat{j} + \hat{k}(-4\lambda)] = 6 \quad \dots(i)$$

On dividing both sides by $\sqrt{(1 + 3\lambda)^2 + (3 - \lambda)^2 + (-4\lambda)^2}$, we get

$$\frac{\vec{r} \cdot [(1 + 3\lambda)\hat{i} + (3 - \lambda)\hat{j} + \hat{k}(-4\lambda)]}{\sqrt{(1 + 3\lambda)^2 + (3 - \lambda)^2 + (-4\lambda)^2}} = \frac{6}{\sqrt{(1 + 3\lambda)^2 + (3 - \lambda)^2 + (-4\lambda)^2}}$$

Since, the perpendicular distance from origin is unity.

$$\begin{aligned} \therefore \frac{6}{\sqrt{(1 + 3\lambda)^2 + (3 - \lambda)^2 + (-4\lambda)^2}} &= 1 \\ \Rightarrow (1 + 3\lambda)^2 + (3 - \lambda)^2 + (-4\lambda)^2 &= 36 \\ \Rightarrow 1 + 9\lambda^2 + 6\lambda + 9 + \lambda^2 - 6\lambda + 16\lambda^2 &= 36 \\ \Rightarrow 26\lambda^2 + 10 &= 36 \\ \Rightarrow \lambda^2 &= 1 \\ \therefore \lambda &= \pm 1 \end{aligned}$$

Using Eq. (i), the required equation of plane is

$$\begin{aligned} \vec{r} \cdot [(1 \pm 3)\hat{i} + (3 \mp 1)\hat{j} + (\mp 4)\hat{k}] &= 6 \\ \Rightarrow \vec{r} \cdot [(1 + 3)\hat{i} + (3 - 1)\hat{j} + (-4)\hat{k}] &= 6 \\ \text{and } \vec{r} \cdot [(1 - 3)\hat{i} + (3 + 1)\hat{j} + 4\hat{k}] &= 6 \\ \Rightarrow \vec{r} \cdot (4\hat{i} + 2\hat{j} - 4\hat{k}) &= 6 \\ \text{and } \vec{r} \cdot (-2\hat{i} + 4\hat{j} + 4\hat{k}) &= 6 \\ \Rightarrow 4x + 2y - 4z - 6 &= 0 \\ \text{and } -2x + 4y + 4z - 6 &= 0 \end{aligned}$$

Q. 25 Show that the points $(\hat{i} - \hat{j} + 3\hat{k})$ and $3(\hat{i} + \hat{j} + \hat{k})$ are equidistant from the plane $\vec{r} \cdot (5\hat{i} + 2\hat{j} - 7\hat{k}) + 9 = 0$ and lies on opposite side of it.

Sol. To show that these given points $(\hat{i} - \hat{j} + 3\hat{k})$ and $3(\hat{i} + \hat{j} + \hat{k})$ are equidistant from the plane $\vec{r} \cdot (5\hat{i} + 2\hat{j} - 7\hat{k}) + 9 = 0$, we first find out the mid-point of the points which is $2\hat{i} + \hat{j} + 3\hat{k}$.

On substituting $\vec{r}$ by the mid-point in plane, we get

$$\begin{aligned} \text{LHS} &= (2\hat{i} + \hat{j} + 3\hat{k}) \cdot (5\hat{i} + 2\hat{j} - 7\hat{k}) + 9 \\ &= 10 + 2 - 21 + 9 = 0 \\ &= \text{RHS} \end{aligned}$$

Hence, the two points lie on opposite sides of the plane are equidistant from the plane.

Q. 26 $\vec{AB} = 3\hat{i} - \hat{j} + \hat{k}$ and $\vec{CD} = -3\hat{i} + 2\hat{j} + 4\hat{k}$ are two vectors. The position vectors of the points A and C are $6\hat{i} + 7\hat{j} + 4\hat{k}$ and $-9\hat{i} + 2\hat{k}$, respectively. Find the position vector of a point P on the line AB and a point Q on the line CD such that $\vec{PQ}$ is perpendicular to $\vec{AB}$ and $\vec{CD}$ both.

Sol. We have, $\vec{AB} = 3\hat{i} - \hat{j} + \hat{k}$ and $\vec{CD} = -3\hat{i} + 2\hat{j} + 4\hat{k}$

Also, the position vectors of A and C are $6\hat{i} + 7\hat{j} + 4\hat{k}$ and $-9\hat{i} + 2\hat{k}$, respectively. Since, $\vec{PQ}$ is perpendicular to both $\vec{AB}$ and $\vec{CD}$.

So, P and Q will be foot of perpendicular to both the lines through A and C .

Now, equation of the line through A and parallel to the vector $\vec{AB}$ is,

$$\vec{r} = (6\hat{i} + 7\hat{j} + 4\hat{k}) + \lambda(3\hat{i} - \hat{j} + \hat{k})$$

and the line through C and parallel to the vector $\vec{CD}$ is given by

$$\vec{r} = -9\hat{j} + 2\hat{k} + \mu(-3\hat{i} + 2\hat{j} + 4\hat{k}) \quad \dots (i)$$

$$\vec{r} = (6\hat{i} + 7\hat{j} + 4\hat{k}) + \lambda(3\hat{i} - \hat{j} + \hat{k})$$

$$\text{and} \quad \vec{r} = -9\hat{j} + 2\hat{k} + \mu(-3\hat{i} + 2\hat{j} + 4\hat{k}) \quad \dots (ii)$$

Let $P(6 + 3\lambda, 7 - \lambda, 4 + \lambda)$ is any point on the first line and Q be any point on second line is given by $(-3\mu, -9 + 2\mu, 2 + 4\mu)$.

$$\begin{aligned} \therefore \vec{PQ} &= (-3\mu - 6 - 3\lambda)\hat{i} + (-9 + 2\mu - 7 + \lambda)\hat{j} + (2 + 4\mu - 4 - \lambda)\hat{k} \\ &= (-3\mu - 6 - 3\lambda)\hat{i} + (2\mu + \lambda - 16)\hat{j} + (4\mu - \lambda - 2)\hat{k} \end{aligned}$$

If $\vec{PQ}$ is perpendicular to the first line, then

$$\begin{aligned} &3(-3\mu - 6 - 3\lambda) - (2\mu + \lambda - 16) + (4\mu - \lambda - 2) = 0 \\ \Rightarrow &-9\mu - 18 - 9\lambda - 2\mu - \lambda + 16 + 4\mu - \lambda - 2 = 0 \\ \Rightarrow &-7\mu - 11\lambda - 4 = 0 \quad \dots (iii) \end{aligned}$$

If $\vec{PQ}$ is perpendicular to the second line, then

$$\begin{aligned} &-3(-3\mu - 6 - 3\lambda) + 2(2\mu + \lambda - 16) + 4(4\mu - \lambda - 2) = 0 \\ \Rightarrow &9\mu + 18 + 9\lambda + 4\mu + 2\lambda - 32 + 16\mu - 4\lambda - 8 = 0 \\ \Rightarrow &29\mu + 7\lambda - 22 = 0 \quad \dots (iv) \end{aligned}$$

On solving Eqs. (iii) and (iv), we get

$$\begin{aligned} &-49\mu - 77\lambda - 28 = 0 \\ \Rightarrow &319\mu + 77\lambda - 242 = 0 \\ \Rightarrow &270\mu - 270 = 0 \\ \Rightarrow &\mu = 1 \end{aligned}$$

Using μ in Eq. (iii), we get

$$\begin{aligned} &-7(1) - 11\lambda - 4 = 0 \\ \Rightarrow &-7 - 11\lambda - 4 = 0 \\ \Rightarrow &-11 - 11\lambda = 0 \\ \Rightarrow &\lambda = -1 \end{aligned}$$

$$\begin{aligned} \therefore \vec{PQ} &= [-3(1) - 6 - 3(-1)]\hat{i} + [2(1) + (-1) - 16]\hat{j} + [4(1) - (-1) - 2]\hat{k} \\ &= -6\hat{i} - 15\hat{j} + 3\hat{k} \end{aligned}$$

Q. 27 Show that the straight lines whose direction cosines are given by $2l + 2m - n = 0$ and $mn + nl + lm = 0$ are at right angles.

Sol. We have, $2l + 2m - n = 0$... (i)
and $mn + nl + lm = 0$... (ii)

Eliminating m from the both equations, we get

$$m = \frac{n - 2l}{2} \quad [\text{from Eq. (i)}]$$

$$\Rightarrow \left(\frac{n - 2l}{2}\right)n + nl + l\left(\frac{n - 2l}{2}\right) = 0$$

$$\Rightarrow \frac{n^2 - 2nl + 2nl + nl - 2l^2}{2} = 0$$

$$\Rightarrow n^2 + nl - 2l^2 = 0$$

$$\Rightarrow n^2 + 2nl - nl - 2l^2 = 0$$

$$\Rightarrow (n + 2l)(n - l) = 0$$

$$\Rightarrow n = -2l \text{ and } n = l$$

$$\therefore m = \frac{-2l - 2l}{2}, m = \frac{l - 2l}{2}$$

$$\Rightarrow m = -2l, m = \frac{-l}{2}$$

Thus, the direction ratios of two lines are proportional to $l, -2l, -2$ and $l, \frac{-l}{2}, l$.

$$\Rightarrow 1, -2, -2 \text{ and } 1, \frac{-1}{2}, 1$$

$$\Rightarrow 1, -2, -2 \text{ and } 2, -1, 2$$

Also, the vectors parallel to these lines are $\vec{a} = \hat{i} - 2\hat{j} - 2\hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$, respectively.

$$\begin{aligned} \therefore \cos \theta &= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \frac{(\hat{i} - 2\hat{j} - 2\hat{k}) \cdot (2\hat{i} - \hat{j} + 2\hat{k})}{3 \cdot 3} \\ &= \frac{2 + 2 - 4}{9} = 0 \end{aligned}$$

$$\therefore \theta = \frac{\pi}{2} \quad \left[\because \cos \frac{\pi}{2} = 0 \right]$$

Q. 28 If $l_1, m_1, n_1, l_2, m_2, n_2$ and l_3, m_3, n_3 are the direction cosines of three mutually perpendicular lines, then prove that the line whose direction cosines are proportional to $l_1 + l_2 + l_3, m_1 + m_2 + m_3$ and $n_1 + n_2 + n_3$ makes equal angles with them.

Sol. Let $\vec{a} = l_1\hat{i} + m_1\hat{j} + n_1\hat{k}$
 $\vec{b} = l_2\hat{i} + m_2\hat{j} + n_2\hat{k}$
 $\vec{c} = l_3\hat{i} + m_3\hat{j} + n_3\hat{k}$
 $\vec{d} = (l_1 + l_2 + l_3)\hat{i} + (m_1 + m_2 + m_3)\hat{j} + (n_1 + n_2 + n_3)\hat{k}$

Also, let α, β and γ are the angles between $\vec{a}$ and $\vec{d}$, $\vec{b}$ and $\vec{d}$, $\vec{c}$ and $\vec{d}$.

$$\begin{aligned} \therefore \cos \alpha &= \frac{l_1(l_1 + l_2 + l_3) + m_1(m_1 + m_2 + m_3) + n_1(n_1 + n_2 + n_3)}{l_1^2 + l_1 l_2 + l_1 l_3 + m_1^2 + m_1 m_2 + m_1 m_3 + n_1^2 + n_1 n_2 + n_1 n_3} \end{aligned}$$

$$\begin{aligned}
 &= (l_1^2 + m_1^2 + n_1^2) + (l_1 l_2 + l_1 l_3 + m_1 m_2 + m_1 m_3 + n_1 n_2 + n_1 n_3) \\
 &= 1 + 0 = 1 \\
 &[\because l_1^2 + m_1^2 + n_1^2 = 1 \text{ and } l_1 \perp l_2, l_1 \perp l_3, m_1 \perp m_2, m_1 \perp m_3, n_1 \perp n_2, n_1 \perp n_3] \\
 \text{Similarly, } \cos \beta &= l_2 (l_1 + l_2 + l_3) + m_2 (m_1 + m_2 + m_3) + n_2 (n_1 + n_2 + n_3) \\
 &= 1 + 0 \text{ and } \cos \gamma = 1 + 0 \\
 \Rightarrow \cos \alpha &= \cos \beta = \cos \gamma \\
 \Rightarrow \alpha &= \beta = \gamma \\
 \text{So, the line whose direction cosines are proportional to } l_1 + l_2 + l_3, m_1 + m_2 + m_3, \\
 n_1 + n_2 + n_3 &\text{ makes equal angles with the three mutually perpendicular lines whose direction} \\
 \text{cosines are } l_1, m_1, n_1, l_2, m_2, n_2 \text{ and } l_3, m_3, n_3 \text{ respectively.}
 \end{aligned}$$

Objective Type Questions

Q. 29 Distance of the point (α, β, γ) from Y-axis is

- (a) β (b) $|\beta|$
 (c) $|\beta| + |\gamma|$ (d) $\sqrt{\alpha^2 + \gamma^2}$

Sol. (d) Required distance $= \sqrt{(\alpha - 0)^2 + (\beta - \beta)^2 + (\gamma - 0)^2} = \sqrt{\alpha^2 + \gamma^2}$

Q. 30 If the direction cosines of a line are k, k and k , then

- (a) $k > 0$ (b) $0 < k < 1$
 (c) $k = 1$ (d) $k = \frac{1}{\sqrt{3}}$ or $-\frac{1}{\sqrt{3}}$

Sol. (d) Since, direction cosines of a line are k, k and k .

$$\therefore l = k, m = k \text{ and } n = k$$

$$\text{We know that, } l^2 + m^2 + n^2 = 1$$

$$\Rightarrow k^2 + k^2 + k^2 = 1$$

$$\Rightarrow k^2 = \frac{1}{3}$$

$$\therefore k = \pm \frac{1}{\sqrt{3}}$$

Q. 31 The distance of the plane $\vec{r} \left(\frac{2}{7} \hat{i} + \frac{3}{7} \hat{j} - \frac{6}{7} \hat{k} \right) = 1$ from the origin is

- (a) 1 (b) 7
 (c) $\frac{1}{7}$ (d) None of these

Sol. (a) The distance of the plane $\vec{r} \left(\frac{2}{7} \hat{i} + \frac{3}{7} \hat{j} - \frac{6}{7} \hat{k} \right) = 1$ from the origin is 1.

[since, $\vec{r} \cdot \vec{n} = d$ is the form of above equation, where d represents the distance of plane from the origin i.e., $d = 1$]



Q. 32 The sine of the angle between the straight line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$

and the plane $2x - 2y + z = 5$ is

- (a) $\frac{10}{6\sqrt{5}}$ (b) $\frac{4}{5\sqrt{2}}$ (c) $\frac{2\sqrt{3}}{5}$ (d) $\frac{\sqrt{2}}{10}$

Sol. (d) We have, the equation of line as

$$\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$$

Now, the line passes through point $(2, 3, 4)$ and having direction ratios $(3, 4, 5)$.

Since, the line passes through point $(2, 3, 4)$ and parallel to the vector $(3\hat{i} + 4\hat{j} + 5\hat{k})$.

$$\therefore \vec{b} = 3\hat{i} + 4\hat{j} + 5\hat{k}$$

Also, the cartesian form of the given plane is $2x - 2y + z = 5$.

$$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k})(2\hat{i} - 2\hat{j} + \hat{k}) = 5$$

$$\therefore \vec{n} = (2\hat{i} - 2\hat{j} + \hat{k})$$

$$\text{We know that, } \sin \theta = \frac{|\vec{b} \cdot \vec{n}|}{|\vec{b}| \cdot |\vec{n}|} = \frac{|(3\hat{i} + 4\hat{j} + 5\hat{k}) \cdot (2\hat{i} - 2\hat{j} + \hat{k})|}{\sqrt{3^2 + 4^2 + 5^2} \cdot \sqrt{4 + 4 + 1}}$$

$$= \frac{|6 - 8 + 5|}{\sqrt{50} \cdot 3} = \frac{3}{15\sqrt{2}} = \frac{1}{5\sqrt{2}}$$

$$\sin \theta = \frac{\sqrt{2}}{10}$$

Q. 33 The reflection of the point (α, β, γ) in the XY -plane is

- (a) $(\alpha, \beta, 0)$ (b) $(0, 0, \gamma)$ (c) $(-\alpha, -\beta, \gamma)$ (d) $(\alpha, \beta, -\gamma)$

Sol. (d) In XY -plane, the reflection of the point (α, β, γ) is $(\alpha, \beta, -\gamma)$.

Q. 34 The area of the quadrilateral $ABCD$ where $A(0, 4, 1)$, $B(2, 3, -1)$, $C(4, 5, 0)$, and $D(2, 6, 2)$ is equal to

- (a) 9 sq units (b) 18 sq units
(c) 27 sq units (d) 81 sq units

Sol. (a) We have, $\vec{AB} = (2-0)\hat{i} + (3-4)\hat{j} + (-1-1)\hat{k} = 2\hat{i} - \hat{j} - 2\hat{k}$

$$\vec{BC} = (4-2)\hat{i} + (5-3)\hat{j} + (0+1)\hat{k} = 2\hat{i} + 2\hat{j} + \hat{k}$$

$$\vec{CD} = (2-4)\hat{i} + (6-5)\hat{j} + (2-0)\hat{k} = -2\hat{i} + \hat{j} + 2\hat{k}$$

$$\vec{DA} = (0-2)\hat{i} + (4-6)\hat{j} + (1-2)\hat{k} = -2\hat{i} - 2\hat{j} - \hat{k}$$

$$\begin{aligned} \therefore \text{Area of quadrilateral } ABCD &= |\vec{AB} \times \vec{BC}| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & -2 \\ 2 & 2 & 1 \end{vmatrix} \\ &= |\hat{i}(-1+4) - \hat{j}(2+4) + \hat{k}(4+2)| \\ &= |3\hat{i} - 6\hat{j} + 6\hat{k}| \\ &= \sqrt{9+36+36} = 9 \text{ sq units} \end{aligned}$$

Q. 35 The locus represented by $xy + yz = 0$ is

- (a) a pair of perpendicular lines
- (b) a pair of parallel lines
- (c) a pair of parallel planes
- (d) a pair of perpendicular planes

Sol. (d) We have, $xy + yz = 0$
 $\Rightarrow xy = -yz$
 So, a pair of perpendicular planes.

Q. 36 If the plane $2x - 3y + 6z - 11 = 0$ makes an angle $\sin^{-1} \alpha$ with X -axis, then the value of α is

- (a) $\frac{\sqrt{3}}{2}$
- (b) $\frac{\sqrt{2}}{3}$
- (c) $\frac{2}{7}$
- (d) $\frac{3}{7}$

Sol. (c) Since, $2x - 3y + 6z - 11 = 0$ makes an angle $\sin^{-1} \alpha$ with X -axis.

$$\vec{b} = (1\hat{i} + 0\hat{j} + 0\hat{k}) \text{ and } \vec{n} = 2\hat{i} - 3\hat{j} + 6\hat{k}$$

We know that, $\sin \theta = \frac{|\vec{b} \cdot \vec{n}|}{|\vec{b}| \cdot |\vec{n}|}$

$$= \frac{|(1\hat{i}) \cdot (2\hat{i} - 3\hat{j} + 6\hat{k})|}{\sqrt{1} \sqrt{4 + 9 + 36}} = \frac{2}{7}$$

Fillers

Q. 37 If a plane passes through the points $(2, 0, 0)$ $(0, 3, 0)$ and $(0, 0, 4)$ the equation of plane is

Sol. We know that, equation of a the plane that cut the coordinate axes at $(a, 0, 0)$ $(0, b, 0)$ and $(0, 0, c)$ is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

Hence, the equation of plane passes through the points $(2, 0, 0)$, $(0, 3, 0)$ and $(0, 0, 4)$ is $\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$.

Q. 38 The direction cosines of the vector $(2\hat{i} + 2\hat{j} - \hat{k})$ are

Sol. Direction cosines of $(2\hat{i} + 2\hat{j} - \hat{k})$ are $\frac{2}{\sqrt{4+4+1}}, \frac{2}{\sqrt{4+4+1}}, \frac{-1}{\sqrt{4+4+1}}$ i.e., $\frac{2}{3}, \frac{2}{3}, \frac{-1}{3}$.

Q. 39 The vector equation of the line $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$ is

Sol. We have, $\vec{a} = 5\hat{i} - 4\hat{j} + 6\hat{k}$ and $\vec{b} = 3\hat{i} + 7\hat{j} + 2\hat{k}$

So, the vector equation will be

$$\vec{r} = (5\hat{i} - 4\hat{j} + 6\hat{k}) + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$$

$$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) - (5\hat{i} - 4\hat{j} + 6\hat{k}) = \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$$

$$\Rightarrow (x-5)\hat{i} + (y+4)\hat{j} + (z-6)\hat{k} = \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$$

Q. 40 The vector equation of the line through the points $(3, 4, -7)$ and $(1, -1, 6)$ is

Sol. We know that, vector equation of a line passes through two points is represented by $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$

$$\text{Here, } \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}, \vec{a} = 3\hat{i} + 4\hat{j} - 7\hat{k}$$

$$\text{and } \vec{b} = \hat{i} - \hat{j} + 6\hat{k}$$

$$\Rightarrow (\vec{b} - \vec{a}) = -2\hat{i} - 5\hat{j} + 13\hat{k}$$

So, the required equation is

$$x\hat{i} + y\hat{j} + z\hat{k} = 3\hat{i} + 4\hat{j} - 7\hat{k} + \lambda(-2\hat{i} - 5\hat{j} + 13\hat{k})$$

$$\Rightarrow (x-3)\hat{i} + (y-4)\hat{j} + (z+7)\hat{k} = \lambda(-2\hat{i} - 5\hat{j} + 13\hat{k})$$

Q. 41 The cartesian equation of the plane $\vec{r} \cdot (\hat{i} + \hat{j} - \hat{k}) = 2$ is

Sol. We have, $\vec{r} \cdot (\hat{i} + \hat{j} - \hat{k}) = 2$

$$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + \hat{j} - \hat{k}) = 2$$

$$\Rightarrow x + y - z = 2$$

which is the required form

True/False

Q. 42 The unit vector normal to the plane $x + 2y + 3z - 6 = 0$ is

$$\frac{1}{\sqrt{14}}\hat{i} + \frac{2}{\sqrt{14}}\hat{j} + \frac{3}{\sqrt{14}}\hat{k}.$$

Sol. True

We have,

$$\vec{n} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\therefore \hat{n} = \frac{\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{1^2 + 2^2 + 3^2}} = \frac{\hat{i}}{\sqrt{14}} + \frac{2\hat{j}}{\sqrt{14}} + \frac{3\hat{k}}{\sqrt{14}}$$

Q. 43 The intercepts made by the plane $2x - 3y + 5z + 4 = 0$ on the coordinate axis are -2 , $\frac{4}{3}$ and $-\frac{4}{5}$.

Sol. True

$$\begin{aligned} \text{We have, } & 2x - 3y + 5z + 4 = 0 \\ \Rightarrow & 2x - 3y + 5z = -4 \\ \Rightarrow & \frac{2x}{-4} - \frac{3y}{-4} + \frac{5z}{-4} = 1 \\ \Rightarrow & \frac{x}{-2} + \frac{y}{\frac{4}{3}} - \frac{z}{\frac{5}{5}} = 1 \\ \Rightarrow & \frac{x}{-2} + \frac{y}{\frac{4}{3}} + \frac{z}{\left(-\frac{4}{5}\right)} = 1 \end{aligned}$$

So, the intercepts are -2 , $\frac{4}{3}$ and $-\frac{4}{5}$.

Q. 44 The angle between the line $\vec{r} = (5\hat{i} - \hat{j} - 4\hat{k}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$ and the plane $\vec{r} \cdot (3\hat{i} - 4\hat{j} - \hat{k}) + 5 = 0$ is $\sin^{-1}\left(\frac{5}{2\sqrt{91}}\right)$.

Sol. False

We have, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{n} = 3\hat{i} - 4\hat{j} - \hat{k}$

Let θ is the angle between line and plane.

$$\begin{aligned} \text{Then, } \sin \theta &= \frac{|\vec{b} \cdot \vec{n}|}{|\vec{b}| \cdot |\vec{n}|} = \frac{|(2\hat{i} - \hat{j} + \hat{k}) \cdot (3\hat{i} - 4\hat{j} - \hat{k})|}{\sqrt{6} \cdot \sqrt{26}} \\ &= \frac{|6 + 4 - 1|}{\sqrt{156}} = \frac{9}{2\sqrt{39}} \\ \therefore \theta &= \sin^{-1} \frac{9}{2\sqrt{39}} \end{aligned}$$

Q. 45 The angle between the planes $\vec{r} \cdot (2\hat{i} - 3\hat{j} + \hat{k}) = 1$ and $\vec{r} \cdot (\hat{i} - \hat{j}) = 4$ is $\cos^{-1}\left(\frac{-5}{\sqrt{58}}\right)$.

Sol. False

We know that, the angle between two planes is given by $\cos \theta = \frac{|\vec{n}_1 \cdot \vec{n}_2|}{|\vec{n}_1| |\vec{n}_2|}$

Here, $\vec{n}_1 = (2\hat{i} - 3\hat{j} + \hat{k})$ and $\vec{n}_2 = (\hat{i} - \hat{j})$

$$\begin{aligned} \therefore \cos \theta &= \frac{|(2\hat{i} - 3\hat{j} + \hat{k}) \cdot (\hat{i} - \hat{j})|}{\sqrt{4 + 9 + 1} \cdot \sqrt{1 + 1}} \\ \Rightarrow \cos \theta &= \frac{|2 + 3|}{\sqrt{14} \cdot \sqrt{2}} = \frac{5}{2\sqrt{7}} \\ \therefore \theta &= \cos^{-1}\left(\frac{5}{2\sqrt{7}}\right) \end{aligned}$$

Q. 46 The line $\vec{r} = 2\hat{i} - 3\hat{j} - \hat{k} + \lambda (\hat{i} - \hat{j} + 2\hat{k})$ lies in the plane

$$\vec{r} \cdot (3\hat{i} + \hat{j} - \hat{k}) + 2 = 0.$$

Sol. False

We have, $\vec{r} = 2\hat{i} - 3\hat{j} - \hat{k} + \lambda (\hat{i} - \hat{j} + 2\hat{k})$

$$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) = \hat{i}(2 + \lambda) + \hat{j}(-3 - \lambda) + \hat{k}(-1 + 2\lambda)$$

Since, $x = (2 + \lambda)$, $y = (-3 - \lambda)$ and $z = (-1 + 2\lambda)$ are coordinates of general point which should satisfy the equation of the given plane.

$$\therefore [(2 + \lambda)\hat{i} + (-3 - \lambda)\hat{j} + (2\lambda - 1)\hat{k}] \cdot [\hat{i} + \hat{j} - \hat{k}] = 2$$

$$\Rightarrow (2 + \lambda) - 3 - \lambda - 2\lambda + 1 = 2$$

$$\Rightarrow -2\lambda = 2$$

$$\Rightarrow \lambda = -1$$

$$\therefore \vec{r} = (2 - 1)\hat{i} + (-3 + 1)\hat{j} + (-2 - 1)\hat{k} \\ = \hat{i} - 2\hat{j} - 3\hat{k}$$

Again, from the equation of the plane

$$\vec{r} \cdot (3\hat{i} + \hat{j} - \hat{k}) + 2 = 0$$

$$\Rightarrow (\hat{i} - 2\hat{j} - 3\hat{k}) \cdot (3\hat{i} + \hat{j} - \hat{k}) + 2 = 0$$

$$\Rightarrow (3 - 2 + 3) + 2 = 0$$

$$\Rightarrow 6 \neq 0$$

which is not true.

So, the line $\vec{r} = 2\hat{i} - 3\hat{j} - \hat{k} + \lambda (\hat{i} - \hat{j} + 2\hat{k})$ does not lie in a plane.

Q. 47 The vector equation of the line $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$ is

$$\vec{r} = 5\hat{i} - 4\hat{j} + 6\hat{k} + \lambda (3\hat{i} + 7\hat{j} + 2\hat{k})$$

Sol. True

We have,

$$x = 5, y = -4, z = 6$$

and

$$a = 3, b = 7, c = 2$$

$$\therefore \vec{r} = (5\hat{i} - 4\hat{j} + 6\hat{k}) + \lambda (3\hat{i} + 7\hat{j} + 2\hat{k})$$

Q. 48 The equation of a line, which is parallel to $2\hat{i} + \hat{j} + 3\hat{k}$ and which passes through the point $(5, -2, 4)$ is $\frac{x-5}{2} = \frac{y+2}{-1} = \frac{z-4}{3}$.

Sol. False

Here,

$$x_1 = 5, y_1 = -2, z_1 = 4$$

and

$$a = 2, b = 1, c = 3$$

$$\Rightarrow \frac{x-5}{2} = \frac{y+2}{1} = \frac{z-4}{3}$$

Q. 49 If the foot of perpendicular drawn from the origin to a plane is $(5, -3, -2)$, then the equation of plane is $\vec{r} \cdot (5\hat{i} - 3\hat{j} - 2\hat{k}) = 38$.

Sol. True

Since, the required plane passes through the point $P(5, -3, -2)$ and is perpendicular to $\vec{OP}$.

$$\therefore \vec{a} = 5\hat{i} - 3\hat{j} - 2\hat{k}$$

$$\text{and } \vec{n} = \vec{OP} = 5\hat{i} - 3\hat{j} - 2\hat{k}$$

Now, the equation of the plane is

$$(\vec{r} - \vec{a}) \cdot \vec{n} = 0$$

$$\Rightarrow \vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$$

$$\Rightarrow \vec{r} \cdot (5\hat{i} - 3\hat{j} - 2\hat{k}) = (5\hat{i} - 3\hat{j} - 2\hat{k}) \cdot (5\hat{i} - 3\hat{j} - 2\hat{k})$$

$$\Rightarrow \vec{r} \cdot (5\hat{i} - 3\hat{j} - 2\hat{k}) = 25 + 9 + 4$$

$$\Rightarrow \vec{r} \cdot (5\hat{i} - 3\hat{j} - 2\hat{k}) = 38$$